Journal of Engineering Physics and Thermophysics, Vol. 81, No. 1, 2008

MATHEMATICAL SIMULATION OF THE DYNAMICS OF A DISPERSED PHASE IN FREE GRAVITATIONAL CONVECTION OF A VISCOUS INCOMPRESSIBLE LIQUID IN A SQUARE CAVITY

L. P. Kholpanov,[a] **E. I. Nekrasova,**[b] **and A. K. Nekrasov**[c]

UDC 66.01/07(075.8):532.533

The problem of motion of monodisperse spherical particles in a heterogeneous medium in nonisothermal free convection of a carrying incompressible liquid in a square cavity with inhomogeneous distribution of temperature on the walls is considered. The problem is solved by the finite-difference method via joint solution of equations for the carrying phase in Euler variables and the equation for a disperse particle in Lagrange variables.

Introduction. In power engineering and in a number of industries (chemical, food, pharmaceutical, metallurgical), the phenomena and problems associated with simulation of inhomogeneous multiphase media play a decisive role. They are currently central in solving a number of environmental problems [1].

In investigation of multiphase inhomogeneous media, most important is the problem of interaction of the relative motion of a dispersed phase with a carrying solid medium. This is achieved by mathematical simulation by numerical solutions, on electronic computers, of stationary and nonstationary problems of hydro- and gas dynamics under the conditions of forced and free-convective motion of heterogeneous media.

Among the various methods of mathematical simulation of the dynamics of heterogeneous media two approaches should be noted.

1. The calculation of the motion of a particle is based on the equation of the dynamics of a material point written in the Lagrangian coordinate system [1, 2]. Here, the influence of the carrying medium is taken into account in terms of its average velocity on the assumption that the trajectory of particles coincides with the direction of the average velocity of flow. However, in a real case the trajectory of the particle cannot coincide with the trajectory of the main flow, since the local components of the tensor of stresses of a two-dimensional swirled flow are inhomogeneous. Moreover, such an approach cannot reproduce the full picture of the trajectory of the particle in a swirled flow. Therein lies the substantial drawback of the approach.

2. The main object of investigation is a continuous carrying medium, whereas a dispersed phase is taken into account in terms of its concentration and the force of interaction between the continuous medium and dispersed phase. The mathematical simulation of the medium is based on the Navier–Stokes type equations. Here, substantial achievements have been attained in this direction [3].

In our opinion, most fruitful is the method detailed out in [4] and based on the combination of these two approaches, that is, the vector equation of motion of a dispersed particle in the Lagrangian coordinate system is solved together with the equations of motion of a continuous carrying medium in the Eulerian coordinate system.

Statement of the Problem. Below we consider the motion of monodisperse particles in a heterogeneous medium in the presence of a nonstationary and stationary nonisothermal free convection of the carrying viscous incompressible liquid in a two-dimensional square cavity of size $L \times L$ with prescribed thermal conditions on its boundaries (see Fig. 1). In order to find the velocity of motion and construct the trajectory of motion of homogeneous dispersed particles in a heterogeneous medium the method presented in [5] was used.

---

[a]Institute of Problems of Chemical Physics, Russian Academy of Sciences, Chernogolovka, Moscow Region, 142432, Russia; email: kholp@icp.ac.ru; [b]Elektrostal Polytechnic Institute (branch of the Moscow State Institute of Steel and Alloys), Russia; [c]Moscow State University of Engineering Ecology, Russia. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 81, No. 1, pp. 81–89, January–February, 2008. Original article submitted October 17, 2006.

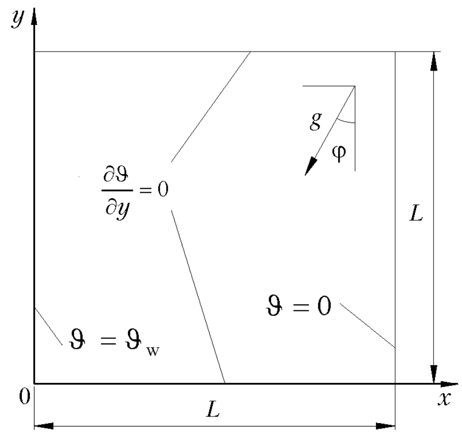

Fig. 1. Computational domain with thermal boundary conditions.

In order to describe the velocity and temperature fields in a laminar flow of a viscous incompressible carrying medium with constant properties, except for the density ρ_1 in an inhomogeneous temperature field, and without account for heat output due to viscous dissipation, the work of compression forces as well as for the influence of particles on the carrying medium the system of the Navier–Stokes equations in the Boussinesq approximation was used:

the equation of motion

$$\frac{\partial V_1}{\partial t} + (\mathbf{V}_1 \nabla)\, \mathbf{V}_1 = -\frac{1}{\rho_1} \nabla p' + \nu \Delta \mathbf{V}_1 + g \beta_t \vartheta \,, \tag{1}$$

the continuity equation

$$\mathrm{div}\, \mathbf{V}_1 = 0 \,, \tag{2}$$

the energy equation

$$\frac{\partial \vartheta}{\partial t} + (\mathbf{V}_1 \nabla)\, \vartheta = a \Delta \vartheta \,. \tag{3}$$

The boundary-value conditions for the system (1)–(3) in a two-dimensional rectangular cavity were taken to be

$$t = 0: \quad \mathbf{V}_1 = 0 \,, \quad \vartheta = 0 \,; \tag{4}$$

$$y = 0 \,; \;\; L \,, \;\; 0 \le x \le L: \;\; \frac{\partial \vartheta}{\partial y} = 0 \,, \;\; \mathbf{V}_{1\mathrm{w}} = 0 \,; \tag{5}$$

$$x = 0 \,, \;\; 0 \le y \le L: \;\; \vartheta = \vartheta_{\mathrm{w}} \,, \;\; \mathbf{V}_{1\mathrm{w}} = 0 \,; \tag{6}$$

$$x = L \,, \;\; 0 \le y \le L: \;\; \vartheta = 0 \,, \;\; \mathbf{V}_{1\mathrm{w}} = 0 \,. \tag{7}$$

Its solution is the velocity field for the carrying medium $\mathbf{V}_1$. The velocity $\mathbf{V}_2$ of a dispersed spherical particle of diameter d and density ρ_2 together with system (1)–(3) is determined by solving the equation of motion of the particle, which in the Lagrangian coordinate system has the form [5]

$$\frac{dV_2}{dt} = -\, k w^2 \mathbf{e} + \mathbf{F} \,, \tag{8}$$

where $k = \frac{3}{4}C\rho_1/(\rho_2 d)$ is the coefficient for a spherical particle, which is related to the resistance force in relative motion of the particle in a viscous fluid. The vector of acceleration of mass forces $\mathbf{F}$ is determined by the sum of the vectors of the external force and Archimedes lifting force. The velocity of the dispersed particle [5] is represented as the sum of the velocities of the solid medium and of the relative velocity of the particle:

$$\mathbf{V}_2 = \mathbf{V}_1 + \mathbf{V}_{rel} , \tag{9}$$

where the components of the velocity vector of the continuous medium $\mathbf{V}_1$ for a two-dimensional case in the Cartesian coordinate system (x, y) are denoted by (U, V), whereas the vector of the relative velocity $\mathbf{V}_{rel}$ is expressed in terms of the modulus of the relative velocity $w = |\mathbf{V}_{rel}|$ and the angle α:

$$\mathbf{V}_{rel} = (w \cos \alpha , \; w \sin \alpha) = w\mathbf{e} . \tag{10}$$

Here, the angle α is equal to the value of the angle of rotation from the coordinate axis x up to the vector $\mathbf{e}$. In the equation of particle motion (8) the velocity of motion of the continuous medium at the same point is taken into account.

In [5] it is shown that to determine the velocity of a particle the vector equation (8) is reduced to two scalar equations with two unknown values w and α with prescribed initial conditions:

$$\frac{dw}{dt} = - kw^2 - (P_x + E_x - F_x) \cos \alpha - (P_y + E_y - F_y) \sin \alpha , \tag{11}$$

$$\frac{d\alpha}{dt} = (P_x + E_x - F_x) \frac{\sin \alpha}{w} - (P_y + E_y - F_y) \frac{\cos \alpha}{w} , \tag{12}$$

$$t = 0 : \quad w = w_0 , \quad \alpha = \alpha_0 , \tag{13}$$

in which the components P_i, E_i, and F_i in the Cartesian coordinate system have the form

$$P_x = U \frac{\partial U}{\partial x} + V \frac{\partial U}{\partial y} , \quad P_y = U \frac{\partial V}{\partial x} + V \frac{\partial V}{\partial y} ,$$

$$E_x = w \left(\frac{\partial U}{\partial x} \cos \alpha + \frac{\partial U}{\partial y} \sin \alpha \right), \quad E_y = w \left(\frac{\partial V}{\partial x} \cos \alpha + \frac{\partial V}{\partial y} \sin \alpha \right),$$

$$F_x = g \left(1 - \frac{\rho_1}{\rho_2} \right) \sin \varphi , \quad F_y = - g \left(1 - \frac{\rho_1}{\rho_2} \right) \cos \varphi .$$

Although in Eq. (8) the interaction of particles with a carrying medium is not presented in an explicit form, it is taken into account in Eqs. (11) and (12) in terms of the local components of the velocity field of the carrying medium U, V and its differential characteristics $\partial/\partial x$ and $\partial/\partial y$, as well as the physicochemical and hydrodynamic characteristics of the particle itself. Therefore, in order to determine the value of velocity and direction of motion of the dispersed particle it is necessary to solve the system of equations (11)–(13) together with the system of equations of motion of the carrying medium (1)–(7).

The resistance coefficient C entering into the parameter k of Eq. (8) for a single spherical particle moving in a carrying medium with a local relative velocity w was taken dependent on the Reynolds number $\mathrm{Re} = wd/\nu$ and was determined from the well-known relations [1]: when $\mathrm{Re} \le 1$, $C = \frac{24}{\mathrm{Re}}$ and when $\mathrm{Re} > 1$, $C = \frac{24}{\mathrm{Re}} (1 + 0.15\mathrm{Re}^{0.687})$. The current coordinates $x(t)$ and $y(t)$ of the position of the dispersed particle in space were found from the equations

$$\frac{dx}{dt} = U + w \cos \alpha , \tag{14}$$

$$\frac{dy}{dt} = V + w \sin \alpha . \tag{15}$$

At the known initial coordinates of the particle x_0 and y_0 and found values of the velocity components of the carrying phase U, V, module w, and of the angle of direction α, for the vector of the relative velocity of the particle the trajectory of the motion of the particle in the space of the carrying medium was calculated from Eq. (14) and (15).

The system of equations (1)–(3), (11)–(13), (14), and (15) with the corresponding boundary-value conditions was solved numerically by the finite-difference method. For a further solution of the problem all the equations and boundary conditions for them were dimensionalized with the aid of the corresponding scales. The scales taken were: the width of the computational domain L for space; L^2/ν for time, ν/L for the velocity, and ϑ_ω for the temperature. Moreover, to exclude the pressure from the number of variables in the equations of motion for a carrying medium the stream function ψ and vortex ω associated with the dimensionless velocity components $\bar{U} = UL/\nu$ and $\bar{V} = VL/\nu$ were introduced by the relations

$$\bar{U} = \frac{\partial\psi}{\partial\bar{y}} , \quad \bar{V} = -\frac{\partial\psi}{\partial\bar{x}} , \quad \omega = \frac{\partial\bar{U}}{\partial\bar{y}} - \frac{\partial\bar{V}}{\partial\bar{x}} . \tag{16}$$

With account for the scales introduced and relations (16), the system of equations (1)–(3) and boundary-value conditions (4)–(7) for the carrying medium in the vortex-stream function variables take the form

$$\frac{\partial\omega}{\partial\tau} + \frac{\partial\psi}{\partial\bar{y}}\frac{\partial\omega}{\partial\bar{x}} - \frac{\partial\psi}{\partial\bar{x}}\frac{\partial\omega}{\partial\bar{y}} = \frac{\partial^2\omega}{\partial\bar{x}^2} + \frac{\partial^2\omega}{\partial\bar{y}^2} - \mathrm{Gr}\left(\frac{\partial\theta}{\partial\bar{x}}\cos\varphi - \frac{\partial\theta}{\partial\bar{y}}\sin\varphi\right), \tag{17}$$

$$\frac{\partial^2\psi}{\partial\bar{x}^2} + \frac{\partial^2\psi}{\partial\bar{y}^2} = \omega , \tag{18}$$

$$\frac{\partial\theta}{\partial\tau} + \frac{\partial\psi}{\partial\bar{y}}\frac{\partial\theta}{\partial\bar{x}} - \frac{\partial\psi}{\partial\bar{x}}\frac{\partial\theta}{\partial\bar{y}} = \frac{1}{\mathrm{Pr}}\left(\frac{\partial^2\theta}{\partial\bar{x}^2} + \frac{\partial^2\theta}{\partial\bar{y}^2}\right), \tag{19}$$

$$\tau = 0 : \quad \psi = \omega = \theta = 0 , \tag{20}$$

$$\bar{x} = 0 , \quad 0 \le \bar{y} \le 1 : \quad \psi = 0 , \quad \frac{\partial\psi}{\partial\bar{x}} = 0 , \quad \theta_{\mathrm{w}} = 1 , \tag{21}$$

$$\bar{x} = 1 , \quad 0 \le \bar{y} \le 1 : \quad \psi = 0 , \quad \frac{\partial\psi}{\partial\bar{x}} = 0 , \quad \theta_{\mathrm{w}} = 0 , \tag{22}$$

$$\bar{y} = 0 ; \quad 1 , \quad 0 \le \bar{x} \le 1 : \quad \psi = 0 , \quad \frac{\partial\psi}{\partial\bar{y}} = 0 , \quad \frac{\partial\theta}{\partial\bar{y}} = 0 . \tag{23}$$

Equations (12)–(15) for a particle with a relative diameter $\bar{d} = d/L$ and the components P_i, E_i, and F_i entering into hem in dimensionless variables with the same reference scales will be represented as

$$\frac{d\overline{w}}{d\tau} = -\overline{k}\overline{w}^2 - (\overline{P}_x + \overline{E}_x - \overline{F}_x)\cos\alpha - (\overline{P}_y + \overline{E}_y - \overline{F}_y)\sin\alpha\,, \tag{24}$$

$$\frac{d\alpha}{d\tau} = (\overline{P}_x + \overline{E}_x - \overline{F}_x)\frac{\sin\alpha}{\overline{w}} - (\overline{P}_y + \overline{E}_y - \overline{F}_y)\frac{\cos\alpha}{\overline{w}}\,, \tag{25}$$

$$\tau = 0: \;\; \overline{w} = \overline{w}_0\,, \;\; \alpha = \alpha_0\,, \tag{26}$$

where

$$\overline{w} = \frac{wL}{\nu}\,; \;\; \overline{k} = kL\,; \;\; \overline{P}_x = \overline{U}\frac{\partial\overline{U}}{\partial\overline{x}} + \overline{V}\frac{\partial\overline{U}}{\partial\overline{y}}\,; \;\; \overline{P}_y = \overline{U}\frac{\partial\overline{V}}{\partial\overline{x}} + \overline{V}\frac{\partial\overline{V}}{\partial\overline{y}}\,;$$

$$\overline{E}_x = \overline{w}\left(\frac{\partial\overline{U}}{\partial\overline{x}}\cos\alpha + \frac{\partial\overline{U}}{\partial\overline{y}}\sin\alpha\right); \;\; \overline{E}_y = \overline{w}\left(\frac{\partial\overline{V}}{\partial\overline{x}}\cos\alpha + \frac{\partial\overline{V}}{\partial\overline{y}}\sin\alpha\right);$$

$$\overline{F}_x = \mathrm{Ga}\left(1 - \frac{\rho_1}{\rho_2}\right)\sin\varphi\,; \;\; \overline{F}_y = -\,\mathrm{Ga}\left(1 - \frac{\rho_1}{\rho_2}\right)\cos\varphi\,; \;\; \mathrm{Ga} = gL^3/\nu^2\,.$$

In the computational domain (Fig. 1) a difference grid of nodes in space was introduced, which was uniform over the coordinate axes, and the system of equations for the carrying phase in the vortex-stream function variables (17)–(18) with boundary-value conditions (20)–(23) was replaced by an implicit locally one-dimensional difference scheme. For nonlinear convective terms in the equation of a vortex (17) the Samarskii monotonic approximation was used. The assigning of the boundary condition for the vortex was made by the method of [6]. The systems of three-diagonal difference equations obtained in this way for the vortex, stream function, and for the temperature were solved by the iteration method of variable directions using the pivot method [7, 8].

In order to calculate and construct the trajectory of the motion of a particle the differential equations (14) and (15) in the finite-difference form can be represented by the relations

$$\overline{x}\,(\tau + \Delta\tau) = \overline{x}\,(\tau) + (\overline{U} + \overline{w}\cos\alpha)\,\Delta\tau\,, \tag{27}$$

$$\overline{y}\,(\tau + \Delta\tau) = \overline{y}\,(\tau) + (\overline{V} + \overline{w}\sin\alpha)\,\Delta\tau\,. \tag{28}$$

The algorithm of the solution of the problem formulated is as follows. First, the system of equations (17)–(19) with boundary-value conditions (20)–(23) is solved for the carrying medium and the distributions of the velocity components are determined at the nodes of the Eulerian grid. After the calculation of the fields of the velocity components $\overline{U}$, $\overline{V}$ in the carrying medium and of their derivatives at each step in time by solving the system of nonlinear equations (24), (25) the module of the relative velocity of particle motion $\overline{w}$ and the angle of direction of the relative velocity vector α was determined at four nodes of the Eulerian grid which are closest to the particle.

Since the position of the dispersed particle in the Lagrangian coordinate system does not coincide with the nodes of the Eulerian difference grid for the carrying medium, prior to the solution of Eqs. (27) and (28) the parameters entering into their right-hand sides were determined with the aid of a linear interpolation polynomial.

In order to solve the problem formulated we have developed an original program on a computer in the algorithmic language Fortran-90 in the Compaq Visual Fortran medium, and this program made it possible to visualize the results of simulation on the screen of a color display (the fields of a vortex, stream function, temperature, velocity vector components and the velocity of the carrying medium, vector field of the velocity of the carrying medium and trajectories of moving particles).

The correctness of the operation of the blocks of the program suggested by us which are intended for calculation of the parameters of the carrying medium have been tested on the problems considered in [7] by comparing the

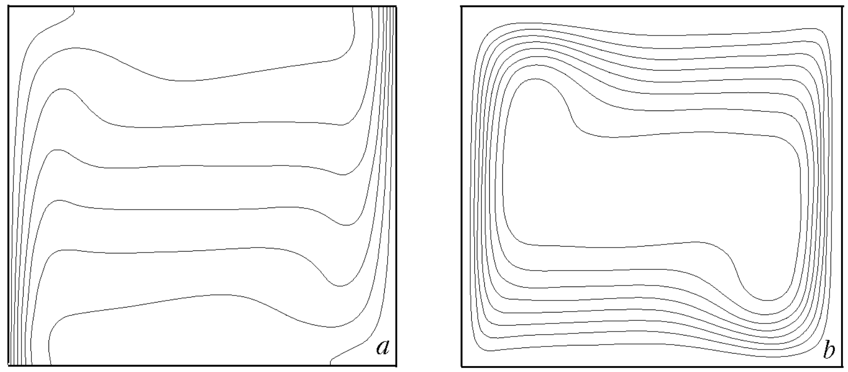

Fig. 2. Isotherms (a) and streamlines (b) for a stationary regime at Gr = 10^6 and Pr = 0.7.

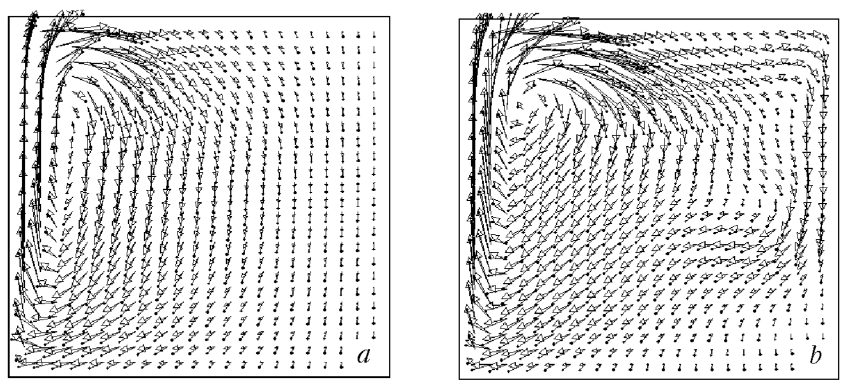

Fig. 3. Nonstationary vector fields of the velocity of the carrying medium (light triangles) and of dispersed particles (dark dots) with relative diameter d = 0.01 and ρ_1/ρ_2 = 0.05 at Ga = $5 \cdot 10^6$ at time moments τ = 0.002 (a) and 0.01 (b).

results at different values of the criteria Gr and Pr. For example, Fig. 2 presents the results of calculation of the thermal gravitational convection of a viscous incompressible liquid in a square cavity, in which the upper and lower walls are thermally insulated and the left wall is more heated than the right one. In the considered range of change in the determining parameters, a satisfactory convergence of the results of calculations was obtained for the maximum values of the stream function, average, maximum and minimum values of the Nusselt number.

In order to simulate the relative motion of dispersed particles in a heterogeneous medium a standard two-dimensional cavity with immobile walls and assigned distribution of temperature on the walls was selected. The cavity is filled with a liquid (or gas) carrying a viscous incompressible medium of density ρ_1 containing monodisperse spherical particles of diameter d with density ρ_2 differing from that of the carrying phase. At the initial time instant the particles are uniformly distributed over the volume of the cavity. Depending on the parameters of the particle and relationship between the forces of viscosity, gravity, Archimedes forces acting on it, and inertia, the particles may settle onto the bottom of the cavity, float up to the upper wall of the cavity, reach the side walls, or remain suspended in the cavity volume, where they are captured by the swirled flow of the carrying medium. It was considered that the particles reaching the walls do not return into the flow; therefore the trajectory of the particles was calculated only till the first contact of the particles with any of the cavity walls. The simulation was made for the cavity located horizontally in space (φ = 0).

Discussion of Results. Analysis of the results obtained has shown that marked changes in the modulus of relative velocity $\overline{w}$ and angle α occur both in the initial period of motion of the heterogeneous medium, which is characterized by substantial nonstationarity of the carrying medium velocity field being formed, and in the period of developed motion of the carrying medium, which is characterized by the considerable inhomogeneity of the velocity field

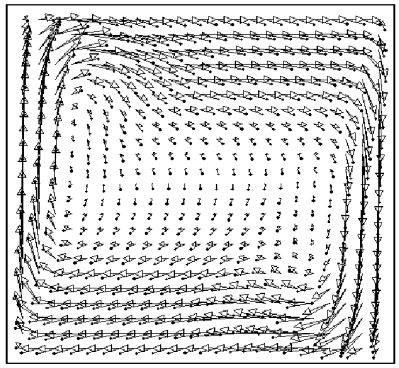

Fig. 4. Developed vector field of the velocity of the carrying medium and of dispersed particles at the parameters given in Fig. 3.

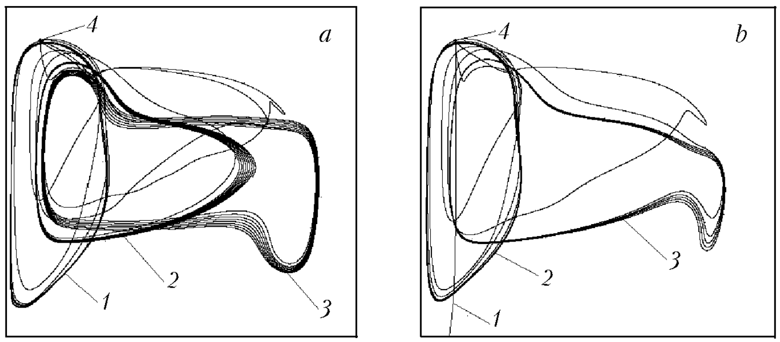

Fig. 5. Trajectories of motion of particles with diameter $\bar{d}$ = 0.0005 and relative density: 1) ρ_1/ρ_2 = 0.001; 2) 0.01; 3) 0.1 at $\bar{x}_0$ = 0.1 and $\bar{y}_0$ = 0.9; Ga = $5{\cdot}10^6$ (a) and $5{\cdot}10^7$ (b).

relative to which the particle is moving. The vector velocity fields at the nodes of the Eulerian grid for homogeneous dispersed particles and for the carrying medium at Ga = $5{\cdot}10^6$, Gr = 10^6, and Pr = 0.7 at different time instants from the beginning of thermal effect on the system are presented in Fig. 3, whereas Fig. 4 demonstrates the established vector field of the velocities of dispersed particles and of the carrying medium.

The vector fields of the velocity of the carrying medium point to the fact that for the initial period of motion of a heterogeneous medium with portions of flow with high velocity gradients there corresponds the mode of a substantially accelerated motion of a particle attributable to a certain (variable in time and spatially inhomogeneous) relationship between the forces of inertia, gravity, Archimedes force, and the force of viscous resistance acting on the particle. The stationary velocity field of the carrying phase is also fairly inhomogeneous; therefore during the motion of the particle in it, just as in unsteady-state motion of the carrying medium, the limiting values of these functions depend on the multitude of parameters.

Below we present some of the results of calculations of the trajectories of motion of disperse particles with different parameters obtained at the following values of the criteria: Gr = 10^6 and Pr = 0.7.

For two values of the Ga number Fig. 5 presents the trajectories of the motion of three heavy ($\rho_1/\rho_2 < 1$) dispersed particles of different density ρ_2 and with the same relative diameter $\bar{d}$ = 0.0005. At the initial instant of time all the particles were located at the point with the same coordinates $\bar{x}_0$ = 0.1 and $\bar{y}_0$ = 0.9 (in all of the subsequent figures the point of the initial position of particles is denoted by the figure 4).

It is seen from Fig. 5 that an increase in the Galileo number because of the increase in the influence of the gravity force, as compared to the viscosity force, raises the relative velocity of the particle prior to the attainment of the viscosity force sufficient for capturing and leads to its rapid settlement (particle 1). In the initial periods of motion of the carrying medium near the heated side of the cavity there is a possibility of capturing heavy particles because of

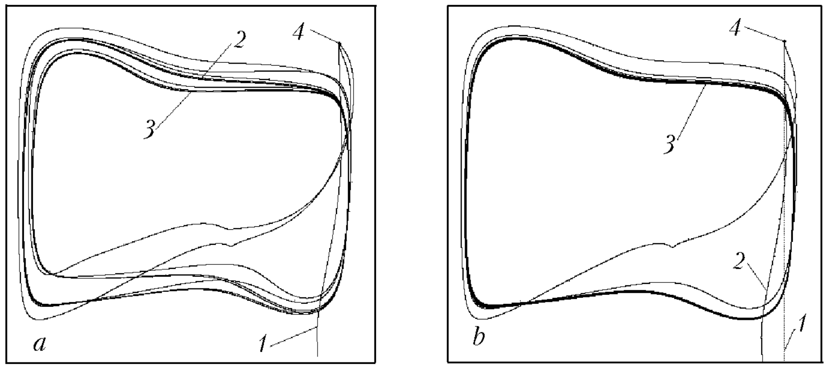

Fig. 6. Trajectories of motion of particles having diameter $\bar{d}$ = 0.0005 and relative density: 1) ρ_1/ρ_2 = 0.001; 2) 0.01; 3) 0.1 at $\bar{x}_0$ = 0.9 and $\bar{y}_0$ = 0.9; Ga = $5 \cdot 10^6$ (a) and $5 \cdot 10^7$ (b).

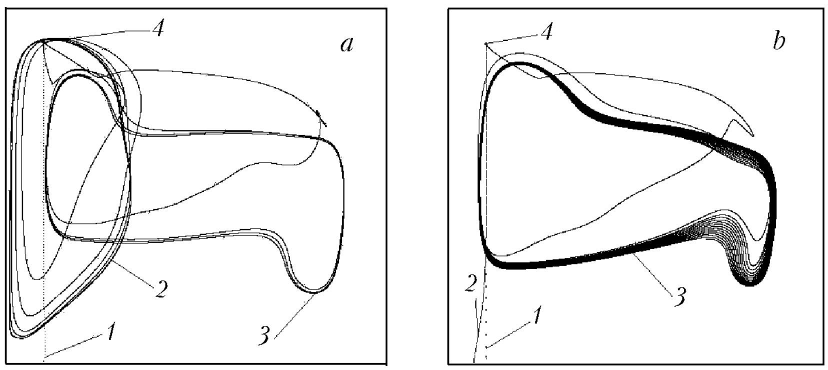

Fig. 7. Trajectories of motion of particles with different diameters (1) $\bar{d}$ = 0.05; 2) 0.005; 3) 0.0005 and relative density ρ_1/ρ_2 = 0.01 at Ga = $5 \cdot 10^6$ (a) and $5 \cdot 10^7$ (b).

the higher relative velocities of the particle and of the carrying medium. As the motion of the carrying medium gets established, its influence on the particle decreases to the extent that the particle settles down under the action of the gravity force.

Figure 6 shows the trajectories of the motion of the same particles, but they begin their motion from the initial position with the coordinates $\bar{x}_0$ = 0.9 and $\bar{y}_0$ = 0.9. It is seen that the initial coordinates of particles substantially influence the character of their motion. Heavy particles which at the initial instants of time are located in the zones of the region far from the heated wall settle down onto the bottom under the gravity force because of the delayed development of the vortex motion of the carrying medium and, as a result, due to the small viscosity forces associated with the value of the relative velocity and insufficient for the capture of the particle by the carrying medium. It is seen from the figure that the same particles captured by the vortex flow may move over a substantially different trajectories than those shown in Fig. 5.

The trajectories of motion for particles with the same relative density ρ_1/ρ_2 = 0.01 and of three different relative diameters $\bar{d}$ for one of the versions of the initial location relative to the heating surface are given in Fig. 7. It is seen that an increase in the diameter of particles at the same relative density ρ_1/ρ_2 leads to an increase in the mass of particles, and to the capture by the flow of the carrying medium only at small values of the Galileo number. Figure 8 shows the trajectories of motion for three particles of different relative density with the same relative diameter $\bar{d}$ = 0.01.

Depending on the ratio of the densities of the continuous medium and dispersed particles both settlement and floating of particles in their motion over trajectories can be observed. Here, a substantial change in the relative velocity and relative angle occurs over those portions of the region of the carrying medium over which a rapid change in

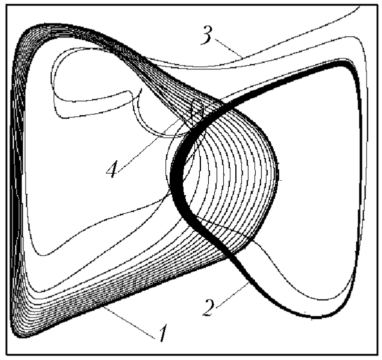

Fig. 8. Trajectories of motion of a heavy (1) ρ_1/ρ_2) and two light (2) ρ_1/ρ_2 = 10; 3) 100) particles of the same relative diameter d = 0.01 at Ga = $5 \cdot 10^6$.

the direction of the motion of the carrying medium flow or acceleration (retardation) of its motion occur. Over the remaining portions of the region the particles move virtually with a constant relative velocity $\overline{w}$ and with the direction angle α.

Calculations have shown that light particles with $\rho_1/\rho_2 >> 1$, when they move in a carrying medium, float up rapidly and reach the upper surface (particle 3 in Fig. 8). It is seen from Fig. 8 that light particle 2, which moves in the zones of the region with a relatively small velocities of the carrying medium, floats up under the action of the Archimedes force, whereas heavy particle 1 with ρ_1/ρ_2 = 0.5 moving in the same zones descends under the action of the gravity force. The motion over the trajectories typical of suspended light and heavy particles (shown in Fig. 8) must lead to an increase in the concentration of heavy particles on the left side of the volume of the region (near the heated surface), whereas the light particles concentrate in the right part of the volume (near the cold surface).

When $\rho_1/\rho_2 < 1$, a decrease in this ratio causes an increase in the relative velocity of the motion of a particle. In this case, the time during which the lower surface of the cavity is reached decreases. In the case of $\rho_1/\rho_2 > 1$, with a decrease in the ratio of the densities, the time of floating up of a particle increases, whereas the value of the relative velocity $\overline{w}$ decreases. The time of floating up or deposition depends on the initial location of the particle. For unsteady motions of the carrying medium the character of the relative motion of a dispersed particle, except for the enumerated above parameters, will depend also on the time of beginning of its motion. In this case the rates of deposition or floating up depend both on the relative density of the particle and on its diameter.

As was expected, with a decrease in the diameter of the particles that have the same density or for dispersed particles with the same diameter and with $\rho_1/\rho_2 \to 1$ the relative velocity, as well as the intensity of deposition or floating up, decrease. With a steady-state regime of the motion of the carrying medium the trajectories of such particles approach in shape to the streamlines with certain values of the stream function. For specific regime parameters of the moving carrying medium (that are determining by the $\underline{\mathrm{G}}$r, Pr, and Ga numbers) dispersed particles with such parameters (the relative density ρ_1/ρ_2 and relative diameter d) may exist which ensure a long suspended motion of a particle together with the carrying medium (floating).

Conclusions. Mathematical simulation of the dynamics of motion of monodispersed spherical particles in nonisothermal free convection of a viscous incompressible carrying fluid in a square cavity is carried out. The finite-difference solution of the problem of motion of a heterogeneous medium is based on the method of paired particles which earlier was applied to the problems with a forced motion.

The results obtained in the work allow the conclusion that the given method can be recommended for mathematical simulation of various hydromechanical processes with heterogeneous working media in both forced and free convection.

NOTATION

a = $\lambda/(c_p\rho_1)$, thermal diffusivity, m^2/sec; C, coefficient of resistance of a spherical particle; c_p, specific heat, J/(kg·K); d, diameter of a dispersed particle, m; **e**, unit vector of the direction of the relative velocity of a particle; **F**, vector of the acceleration of mass forces; Ga = gL^3/ν^2, Galileo number; Gr = $\mathrm{Ga}\cdot\vartheta\beta_t$, Grashof number; g, free fall acceleration, m/sec^2; L, characteristic dimension (width of the cavity), m; Pr = ν/a, Prandtl number; p', relative pressure, Pa; t, time, sec; U, component of the velocity vector of a continuous medium in the direction of the x axis, m/sec; V, component of the velocity vector of a continuous medium in the direction of the y axis, m/sec; **V**, velocity vector; w, module of the relative velocity of a particle, m/sec; x, y, spatial coordinates, m; α, angle of the rotation from the axis x up to the vector **e**, rad; β_t, coefficient of thermal expansion of the carrying medium, 1/K; $\theta = \vartheta/\vartheta_w$, dimensionless temperature; ϑ, relative temperature, K; λ, thermal conductivity, W/(m·K); ν, coefficient of the kinematic viscosity of the carrying phase, m^2/sec; ρ, density, kg/m^3; $\tau = t\nu/L^2$, dimensionless time; φ, angle of inclination of the vector of mass forces relative to the vertical, rad; ω, vortex; φ, stream function. Subscripts: rel, relative; t, thermal; w, wall; 0, initial parameters; 1, carrying medium; 2, particle. Superscripts: bar over a symbol, dimensionless quantity.

REFERENCES

1. P. C. Reist, *Aerosol Science and Technology* [Russian translation], Mir, Moscow (1987).
2. D. V. Lyubimov, T. P. Lyubimova, and A. V. Shtaube, Capture of dust particles by a convective vortex, in: *Proc. 3rd Russian National Conf. on Heat Transfer* [in Russian], Vol. 5, MÉI, Moscow (2002), pp. 258–261.
3. R. I. Nigmatulin, *Principles of the Mechanics of Heterogeneous Media* [in Russian], Nauka, Moscow (1978).
4. L. P. Kholpanov, B. R. Ismailov, and P. Vlasak, Modelling of multiphase flow containing bubbles, drops and solid particles, *Eng. Mech.*, **12**, No. 6, 1–11 (2005).
5. L. P. Kholpanov and R. I. Ibyatov, Mathematical simulation of the dynamics of a dispersed phase, *Teor. Osnovy Knim. Tekhnol.*, **39**, No. 2, 206–215 (2005).
6. V. I. Polezhaev and V. L. Gryaznov, Method of calculation of boundary conditions for the Navier–Stokes equations in "vortex, stream function" variables, *Dokl. Akad. Nauk SSSR*, **219**, No. 2, 30–34 (1974).
7. V. M. Paskonov, V. I. Polezhaev, and L. A. Chudov, *Numerical Simulation of Heat- and Mass Transfer Processes* [in Russian], Nauka, Moscow (1984).
8. V. I. Polezhaev, A. V. Buné, N. I. Verezub, et al., *Mathematical Simulation of Convective Heat and Mass Transfer on the Basis of Navier–Stokes Equations* [in Russian], Nauka, Moscow (1987).